

# QMC with large CI trial wave functions

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# Wave function methods

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## Chlorine atom in cc-pVDZ

- 9  $\uparrow$  and 8  $\downarrow$  electrons in 19 MOs
- Full-CI: 92378  $\uparrow$  determinants and 75582  $\downarrow$  determinants
- $6.98 \cdot 10^9$  possible Slater determinants
- The size of the Full-CI space is huge, but the space is **empty** :  
 $\sim 10^6$  coefficients  $|c_k| > 10^{-12}$

$\implies$  Selected CI algorithm to select iteratively the most important Slater determinants

# Brief history of Selected CI

- 1969 Bender / Davidson *et al* : one-shot selection with perturbative criterion
- 1969 Whitten / Ackmeyer : Independently proposed the same criterion, but with iterative selection
- 1973 Malrieu *et al* : Add a PT2 calculation to the selected determinants (CIPSI)

Groups using/developing CIPSI independently since then

- Angeli, Cimiraglia, Persico : Italy
- Barone : Italy
- Illas, Rubio, Ricart : Spain
- Malrieu, Daudey, Spiegelman : France

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Iterative selected CI has been re-invented many times.



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MRDCI (< 1980)



Adaptative CI

(Evangelista,2014)

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Iterative CI (Liu, 2016)



All differ by the selection  
criterion and the implementation.

but they refer to the *same*  
*natural idea*

# Selected CI algorithm

1. Define a *reference* wave function:

$$|\Psi\rangle = \sum_{i \in \mathcal{D}} c_i |i\rangle \quad E_{\text{var}} = \frac{\langle \Psi | \hat{H} | \Psi \rangle}{\langle \Psi | \Psi \rangle}$$

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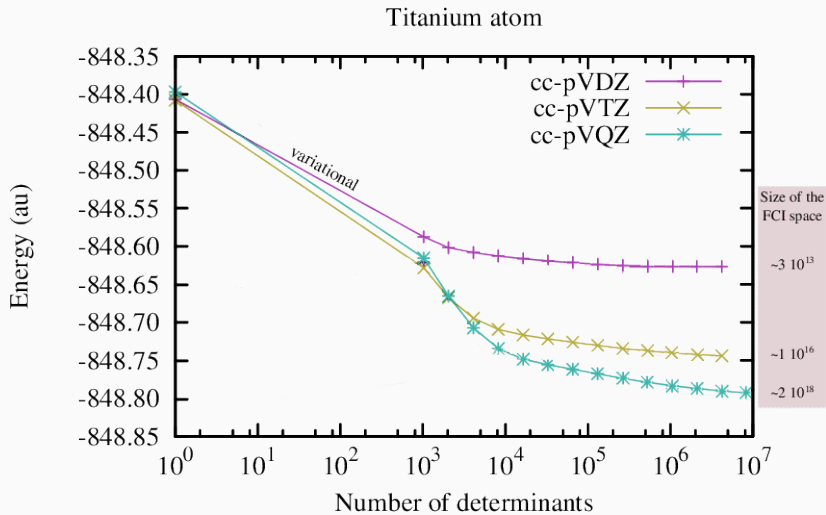
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6. Iterate

# Illustration

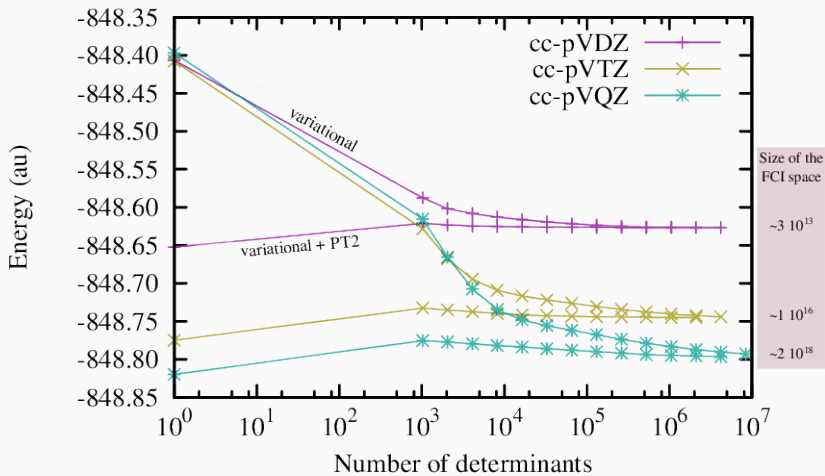


# Remarks

- When all  $|\alpha\rangle$ 's are selected the Full-CI is obtained
- CIPSI is more an *algorithm* than a *method*
- CIPSI can be seen as the deterministic counterpart of FCIQMC
- Any WF method can be realized with the CIPSI algorithm
- Rules on the generation of  $|\alpha\rangle$ 's define the wave function method (CISD, CAS, MRCI, ...)
- Rules on the selection of  $|\alpha\rangle$ 's define the person you need to cite for the trademark
- With the CIPSI selection, at any time  $E_{PT2} = \sum_{\alpha} \Delta E_{\alpha}$  estimates the distance to the solution

# Illustration

## Titanium atom



# Making CIPSI efficient

1. Initiator-like approximation : Generate only  $\alpha$  determinants from the most important determinants (typically 99% of the norm) (Evangelisti, Chem. Phys., 1983). Typically  $\sim 2\,000$  generators for  $10^7$  determinants
2. Each  $\alpha$  determinant is connected to only a subset of determinants of the wave function  $\implies \Delta E_\alpha$  is done in *constant time* (Cimiraglia, JCP, 1985)
3. Evaluation of  $H_{\alpha\alpha}$  needs 10 flops in the worst case : doesn't scale with the number of electrons (Cimiraglia, JCP, 1985)

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Our contributions:

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Our contributions:

1. Efficient implementation of Slater Condon's rules (ArXiv 2013)
2. Knowing that  $H_{ij}$  is zero can be done in 5.3 CPU cycles,  $\sim 50\times$  faster than a random memory access
3. Sorting :  $\mathcal{O}(N \log(N))$  for mathematicians but  $\mathcal{O}(N)$  for computer scientists (radix sort algorithm)

# Making CIPSI efficient

- Selection and PT2 scale *linearly* with the number of variational determinants
- Efficient hybrid stochastic PT2<sup>1</sup> which converges to the exact (zero error) value in *finite* time. Error convergence rate is  $\mathcal{O}(1/t^{3.5})$ .
- Selection is embarrassingly parallel (client/server task parallelism)
- Practical bottleneck is today the Davidson diagonalization (work in progress)

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<sup>1</sup>Hybrid stochastic-deterministic calculation of the MRPT2

Y. Garniron, A. Scemama, P.-F. Loos, M. Caffarel, *J. Chem. Phys.*, **147**, 034101, (2017).



# What we can do today

Big picture: we can address the same problems as other selected CI methods, including FCI-QMC.

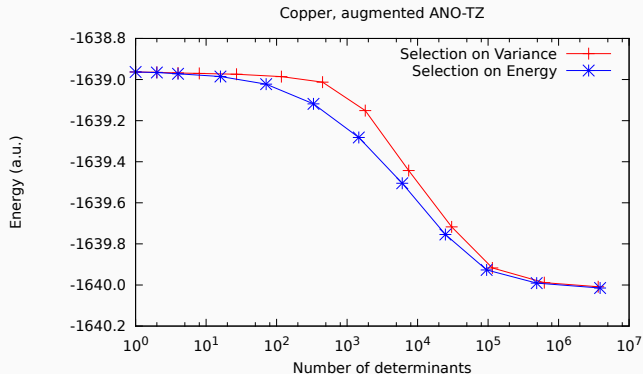
These are *not* heroic runs but large runs:

- $\text{Cr}_2$  cc-pVQZ, 20 million dets + PT2, (JCP 2017)
- Butadiene, 60 million dets+PT2 (work in progress)
- $\text{FeO}_4$ , def2-TZVPP, 50 million dets +PT2 (work in progress)
- $\text{Cu}_2\text{O}_2^{2+}$ , 6-31G benchmark (work in progress)
- We could not do more than 60 million because of a stupid 32-bit integer limit in a library call (will be fixed soon)

# CIPSI vs HCI

$$\text{Energy} : \sum_{\alpha \notin \mathcal{D}} \frac{\langle \Psi | \hat{H} | \alpha \rangle \langle \alpha | \hat{H} | \Psi \rangle}{E_{\text{var}} - \langle \alpha | \hat{H} | \alpha \rangle}$$

$$\text{Variance} : \sum_{\alpha \notin \mathcal{D}} \langle \Psi | \hat{H} | \alpha \rangle \langle \alpha | \hat{H} | \Psi \rangle$$



Difference:

CIPSI : Energy minimization

HCI : Variance minimization

QMC

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# Jastrow factors?

- Full-CI : post-Hartree-Fock method

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*What else?*

So in this talk, I have used my favorite Jastrow factor<sup>2</sup>:

$$J = 1$$

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# Efficient scheme for CI wave functions in QMC

$\Psi$  can be expressed in a bi-linear form<sup>3</sup>:

$$\Psi(\mathbf{R}) = \sum_k^{N_{\text{det}}} c_k \mathcal{D}_k = \sum_i^{N_{\text{det}\uparrow}} \sum_j^{N_{\text{det}\downarrow}} C_{ij} D_{i\uparrow}(\mathbf{R}_{\uparrow}) D_{j\downarrow}(\mathbf{R}_{\downarrow}) \quad (1)$$

- $\mathbf{D}_{\uparrow}(\mathbf{R}_{\uparrow})$  : vector of  $N_{\text{det}\uparrow}$  elements
- $\mathbf{D}_{\downarrow}(\mathbf{R}_{\downarrow})$  : vector of  $N_{\text{det}\downarrow}$  elements
- $\mathbf{C}$  :  $N_{\text{det}\uparrow} \times N_{\text{det}\downarrow}$  matrix. The matrix contains  $N_{\text{det}}$  elements.

$\mathbf{C}$  is constant in a QMC run  $\implies$  preprocessing.

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<sup>3</sup>QMC with very large multideterminant wavefunctions  
*J. Comput. Chem.*, **37:20**, 1866–1875, (2016).



# Efficient scheme for CI wave functions in QMC

At every MC step, we need to evaluate: ( $\uparrow$  electrons and  $\downarrow$  electrons)

$$\Psi = (\mathbf{D}_{\uparrow}^{\dagger}(\mathbf{C})\mathbf{D}_{\downarrow}) \quad (2)$$

$$\nabla_i \Psi = \nabla_i \mathbf{D}_{\uparrow}^{\dagger} \cdot (\mathbf{C}\mathbf{D}_{\downarrow}) \text{ or } (\mathbf{D}_{\uparrow}^{\dagger} \mathbf{C}) \cdot \nabla_i \mathbf{D}_{\downarrow} \quad (3)$$

$$\Delta_i \Psi = \Delta_i \mathbf{D}_{\uparrow}^{\dagger} \cdot (\mathbf{C}\mathbf{D}_{\downarrow}) \text{ or } (\mathbf{D}_{\uparrow}^{\dagger} \mathbf{C}) \cdot \Delta_i \mathbf{D}_{\downarrow} \quad (4)$$

$$V_{\text{pseudo}}^{\text{non-loc}} \Psi = V_{\text{pseudo}}^{\text{non-loc}} \mathbf{D}_{\uparrow}^{\dagger} \cdot (\mathbf{C}\mathbf{D}_{\downarrow}) \text{ or } (\mathbf{D}_{\uparrow}^{\dagger} \mathbf{C}) \cdot V_{\text{pseudo}}^{\text{non-loc}} \mathbf{D}_{\downarrow} \quad (5)$$

- Only  $\Psi$  requires  $\mathcal{O}(N_{\text{det}})$  operations (tiny prefactor)
- Others are  $\mathcal{O}(N_{\text{elec}\uparrow} \times N_{\text{det}\uparrow}) \implies$  expensive work is  $\mathcal{O}(\sqrt{N_{\text{det}}})$

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- Others are  $\mathcal{O}(N_{\text{elec}\uparrow} \times N_{\text{det}\uparrow}) \implies$  expensive work is  $\mathcal{O}(\sqrt{N_{\text{det}}})$
- **We can now use  $\sim 2\,000\,000$  determinants in DMC.**

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Back 30 years ago<sup>4</sup>:

## C. ECP-QMC

For QMC we render the nonlocal ECP operator in a local form. This cannot in general be done exactly,<sup>17</sup> but to a very good approximation this can be accomplished simply by allowing  $U^{\text{ECP}}$  to act on  $\Psi_{\text{val}}$  as implied in Eq. (6b). This leads to an additional term in the local energy, namely

$$U_{\text{Local}}^{\text{ECP}} = \sum_A \sum_{l=1}^{N_{\text{val}}} \left( U_{l_{\text{max}}+1}^A(r_{iA}) + \sum_{l=0}^{l_{\text{max}}} \sum_{m=-l}^l Y_{lm}(\Omega_{iA}) \right. \\ \left. \times U_l^A(r_{iA}) \langle Y_{lm}(\Omega_{iA}) | \Psi_{\text{val}} \rangle / \Psi_{\text{val}} \right), \quad (10)$$

where  $\Psi_{\text{val}}$  becomes the QMC valence importance function.

$$\langle Y_{lm}(\Omega_{iA}) | \Psi_{\text{val}} \rangle / \Psi_{\text{val}} = \sum_j D_{ji}^{-1} \langle Y_{lm}(\Omega_{iA}) | \phi_j(i) \rangle, \quad (11)$$

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<sup>4</sup>Valence quantum Monte Carlo with abinitio effective core potentials  
B. Hammond, *et al*, *J. Chem. Phys.*, **87**:2, 1130–1136, (1987).

# Pseudo-potentials

- $\langle Y_{lm}(\Omega_{iA}) | \Phi_j(i) \rangle$  can be computed **analytically**
- For efficiency: pre-computed on a grid
- No more quadrature points to compute
- No more need for “T-moves or not T-moves”
- Cost : Same as Laplacian ( $\sim 15 - 20\%$  of a MC step)
- Of course, also applicable to single-determinant for DFT trial wave functions

## Some results

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# 1. FCI/DMC — H<sub>2</sub>O

## All-electron quasi-Full-CI trial wave functions:

TABLE I. Number of determinants and corresponding variational energies for CIPSI expansions used in DMC for each cc-pCVnZ (n = 2 to 5) basis set. Last column: Deviations of the variational energy to the best FCI estimates of Almora-Diaz.<sup>27</sup> Energies in atomic units.

Basis set	FCI size	# dets used in DMC	$E_0^{var}$	FCI, Almora-Diaz <sup>27</sup>	Deviation
cc-pCVDZ	$\sim 10^{10}$	172 256	-76.282 136	-76.282 865	0.0007
cc-pCVTZ	$\sim 2 \cdot 10^{14}$	640 426	-76.388 287	-76.390 158	0.0018
cc-pCVQZ	$\sim 2 \cdot 10^{17}$	666 927	-76.419 324	-76.421 148	0.0018
cc-pCV5Z	$\sim 7 \cdot 10^{19}$	1 423 377	-76.428 550	-76.431 105	0.0025

Basis set[Ndets]	T <sub>CPU</sub> (Ndets)/T <sub>CPU</sub> (1 det)	$E_0^{DMC}$
cc-pCVDZ[172 256]	$\sim 101$	-76.415 71(20)
cc-pCVTZ[640 426]	$\sim 185$	-76.431 82(19)
cc-pCVQZ[666 927]	$\sim 128$	-76.436 22(14)
cc-pCV5Z[1 423 377]	$\sim 235$	-76.437 44(18)

<sup>4</sup>Toward an improved control of the FN error in QMC: The case of the water molecule, *J. Chem. Phys.*, **144**, 151103, (2016).

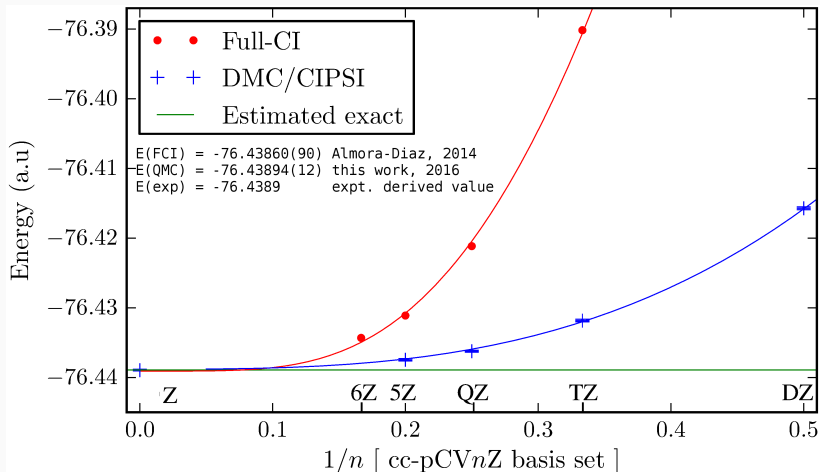
# 1. FCI/DMC — H<sub>2</sub>O

TABLE III. Comparison of nonrelativistic ground-state total energies of water obtained with the most accurate theoretical methods. Energies in a.u.

Clark <i>et al.</i> , <sup>20</sup> DMC (upper bound)	-76.436 8(4)
This work, DMC (upper bound)	-76.437 44(18)
Almora-Dìaz, <sup>27</sup> CISDTQQnSx (upper bound)	-76.434 3
Helgaker <i>et al.</i> , <sup>29</sup> R12-CCSD(T)	-76.439(2)
Muller and Kutzelnigg, <sup>30</sup> R12-CCSD(T)	-76.437 3
Almora-Dìaz, <sup>27</sup> FCI + CBS	-76.438 6(9)
Halkier <i>et al.</i> , <sup>31</sup> CCSD(T) + CBS	-76.438 6
Bytautas and Ruedenberg, <sup>32</sup> FCI + CBS	-76.439 0(4)
This work, DMC + CBS	-76.438 94(12)
Experimentally derived estimate <sup>25</sup>	-76.438 9

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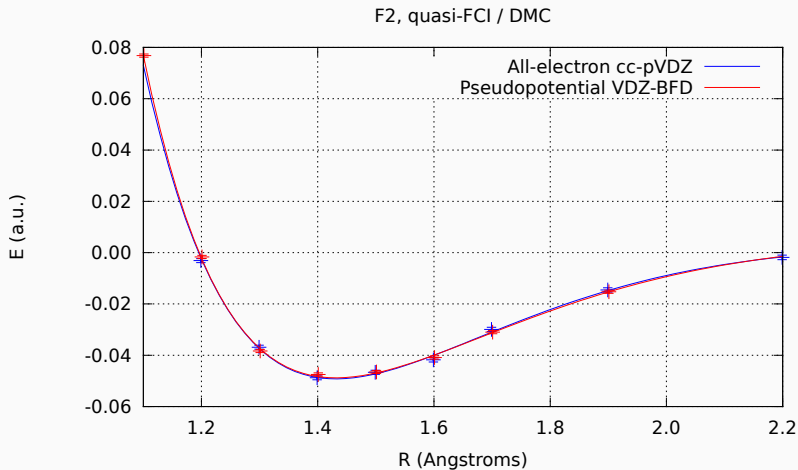
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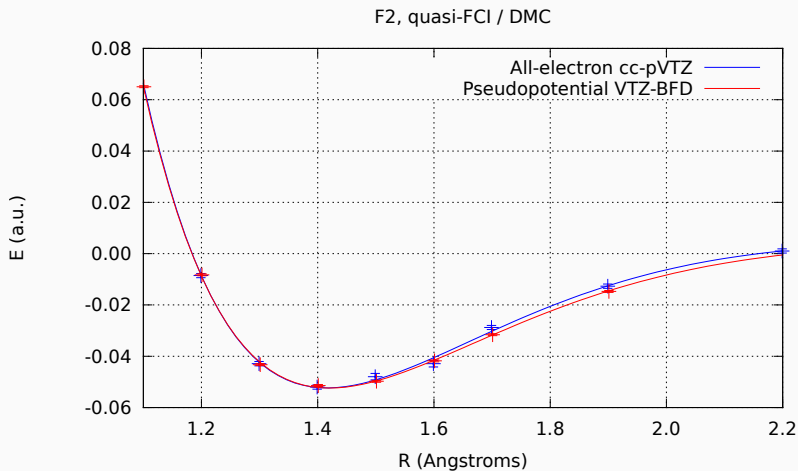
## 2. Pseudopotentials — C<sub>2</sub>

C <sub>2</sub> , cc-pVTZ and VTZ-BFD					
		Energy		Number of determinants	
	C (a.u.)	C <sub>2</sub> (a.u.)	AE (kcal/mol)	C	C <sub>2</sub>
Hartree-Fock					
all-e	-37.6867	-75.4015	17.6	1	1
pseudo-	-5.3290	-10.6880	18.8	1	1
CIPSI					
all-e	-37.7810	-75.7852	140.1	3796	10 <sup>6</sup>
pseudo-	-5.4280	-11.0800	140.6	3882	10 <sup>6</sup>
DMC-HF					
all-e	-37.8293(1)	-75.8597(3)	126.3(2)	1	1
pseudo-	-5.4167(1)	-11.0362(3)	127.2(2)	1	1
DMC-CIPSI, $\epsilon = 10^{-6}$					
all-e	-37.8431(2)	-75.9166(2)	144.6(2)	3497	173553
pseudo-	-5.4334(1)	-11.0969(3)	144.3(2)	3532	231991
Estimated exact AE			147±2		

## 2. Pseudopotentials — $F_2$



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This effect is responsible for some of the *Localization Error*. A scheme for very large systems (low-quality Jastrow + Kohn-Sham determinant)

- Kohn-Sham determinant
- Use a Jastrow to reduce the fluctuations
- Localize the pseudopotentials only on the determinantal component
- The FN-DMC energy will no longer depend on the Jastrow, similarly to all-electron calculations
- Additivity of energies is conserved in H whatever the Jastrow
- No quadrature points to compute

### 3. Dissociation of FeS

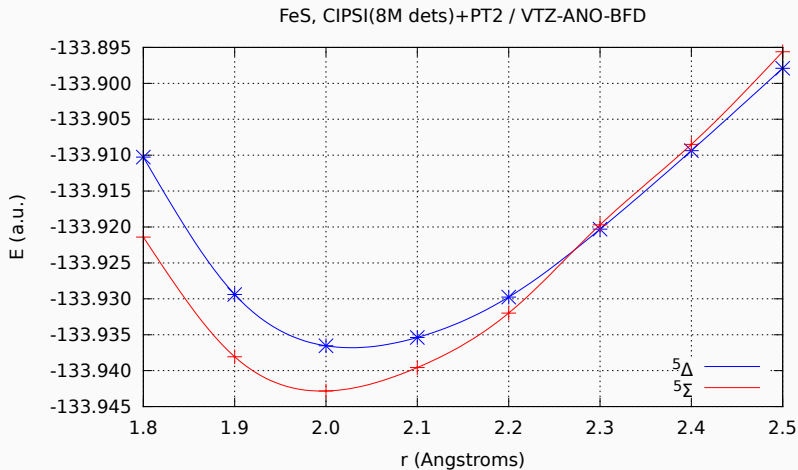
Recent work of Mood and Lüchow<sup>5</sup>:

- Experiment:  $^5\Delta$  ground state
- DFT : mostly  $^5\Delta$ , some  $^5\Sigma$ , Post-HF :  $^5\Sigma$
- Single determinant DMC gives inaccurate results
- Full optimization (Jastrow, CI, MOs) + FN-DMC confirms the  $^5\Delta$  ground state
- What do we get with CIPSI?
- What do we get with CIPSI+DMC?

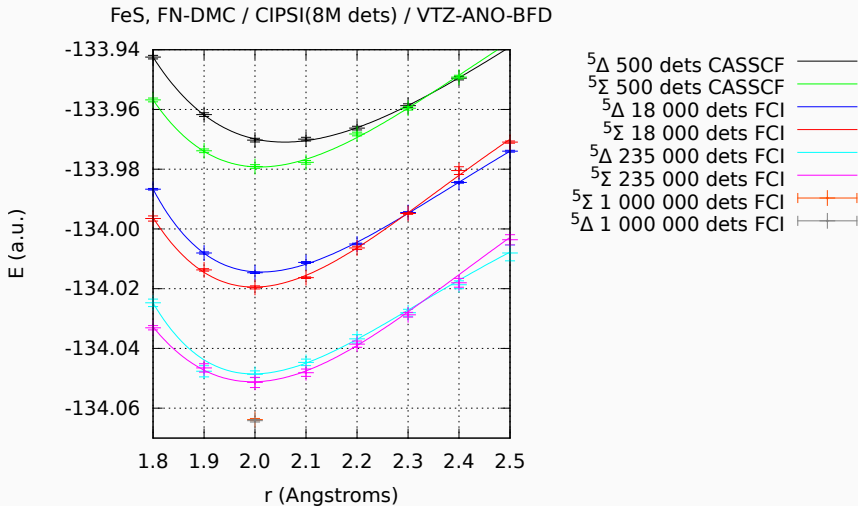
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<sup>5</sup>Full WF optimization with QMC and its effect on the dissociation energy of FeS, K. H. Mood, A. Lüchow, *ArXiv*[physics.chem-ph]: 1702.06535v3

### 3. Dissociation of FeS



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	Mood/Lüchow		This work	Exp.
Energies		~ 1M dets	Extrapolated to 8M	
$^5\Sigma$	-134.0571(4)	-134.0638(4)	-134.0696(6)	
$^5\Delta$	-134.0579(4)	-134.0642(4)	-134.0714(6)	
Fe	-123.8126(4)	-123.8321(4)		
S	-10.1314(1)	-10.1325(3)		
$D_0$ (eV)	3.159(15)	2.770(17)	2.965(21)	3.25(15)
Frequencies		18 000	235 000 dets	
$^5\Sigma$	518(7)	558(25)	535(38)	
$^5\Delta$	499(11)	542(37)	544(31)	518(5)
Equilibrium		18 000	235 000 dets	
$^5\Sigma$	2.00(1)	1.9882(3)	1.9933(4)	
$^5\Delta$	2.031(7)	2.0124(2)	1.9909(7)	2.017

# Summary

## 1. quasi-Full-CI wave functions:

- DMC can be used as a “post-Full-CI” method
- Black-box : no need to define a CAS
- Results are reproducible and well defined
- FCI is orbital-invariant and size-consistent
- Fixed-node error is systematically improvable (Water)
- Sensitivity to the basis set: atomization energies are always underestimated because atoms are described better than molecules, but less severely than in FCI

## 2. Pseudopotentials:

- No numerical integration needed
- No need for T-moves
- Size-consistent wave functions : localization error behaves well ( $F_2$ , FeS)

### 3. What can we do to reduce the dependence on the basis set?

- CIPSI+Jastrow (E. Giner *et al*). See Anouar's talk on Friday.
- Another old idea : basis functions at the middle of bonds
- CI with range-separated DFT (E. Giner). LDA can play the role of the Jastrow. Preliminary results on  $F_2$  increase the atomization energy by 2 kcal/mol:

R(Å)	CIPSI	LDA/CIPSI
4.0	-199.437(1)	-199.437(1)
1.4	-199.4867(6)	-199.4899(6)

- Combining Selected CI with  $f_{12}$  (P. F. Loos) : very promising results
- These schemes are applicable to any post-HF method

## 4. What can we do for larger systems?

Quasi-Full-CI wave functions are too large. We can apply the selected CI algorithm to:

- CAS+SD: not size-consistent but very good nodes (careful with pseudos)
- MR-CCSD(T)<sup>6</sup>/ MR-(SC)2: PhD. thesis of Y. Garniron
- DD-CI: Excited states, magnetic systems
- $B_k$  method method (Davidson *et al* 1981): Dress the Hamiltonian with the PT2  $\implies$  diagonalize in the presence of the  $|\alpha\rangle$
- JM-Heff-PT2: Effective Hamiltonian with a size-consistent MR-PT2 scheme (Giner, JCP 2017)
- Any other post-HF method

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<sup>6</sup>Alternative definition of excitation amplitudes in state-specific MRCC, Y. Garniron, E. Giner, J.-P. Malrieu, A. Scemama *J. Chem. Phys.*, **146:15**,

All calculations performed with our codes (open source, GitHub) :

- Quantum Package (Selected CI)
- QMC=Chem

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